USING THE ELASTICITY OF A FUNCTION IN THE SOCIAL AREA

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Abstract

The paper presents some ascpecte on how to use the elasticity of social functions, using optimal consumer relationship based on Lagrange multiplier type. Also are some considerations concerning the interpretation of Lagrange multiplier and analysis functions request or demand elasticity.

1. Optimal relationship. Lagrange multipliers

The problem of optimal consumer is one subject that solves extreme by introducing a Lagrange multiplier and optimization of next function:

$$L(q_1, q_2, \lambda) = U(q_1, q_2) - \lambda(p_1 q_1 + p_2 q_2 - V).$$

The maximum value $\max\limits_{q_1, q_2, \lambda} L(q_1, q_2, \lambda)$ entails imposition of I the following conditions:

$$\frac{\partial L}{\partial q_1} = 0 \Rightarrow \frac{\partial U}{\partial q_1} = \lambda p_1 \quad (1)$$

$$\frac{\partial L}{\partial q_2} = 0 \Rightarrow \frac{\partial U}{\partial q_2} = \lambda p_2 \quad (2)$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow p_1 q_1 + p_2 q_2 = V \quad (3)$$

Dividing relations (1) and (2) obtain optimal condition:

$$\frac{\partial U}{\partial q_1} : \frac{\partial U}{\partial q_2} = \frac{p_1}{p_2} \Rightarrow \frac{Umg_{q_1}}{p_1} = \frac{Umg_{q_2}}{p_2} \quad (4),$$

where Umg_{q_1} is based on the marginal utility of consumption good 1 (the more the consumer's utility when he increased the consumption of 1 common monetary unit).

Consider the example of the utility function $U(q_1, q_2) = \sqrt{q_1 q_2}$. Optimum condition (4) is written using the utility function as follows:

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$$\frac{\frac{\sqrt{q_2}}{2\sqrt{q_1}}}{p_1} = \frac{\frac{\sqrt{q_1}}{2\sqrt{q_2}}}{p_2} \Rightarrow q_2 = \frac{p_1}{p_2} q_1 \qquad (5)$$

Substituting equation (5) the restriction - the relation (3) - get the optimal amount consumed of good 1:

$$q_1^* = \frac{V}{2p_1} = f_1(p_1, p_2, V) \quad (6)$$

This optimal point is good demand for one type Marshal made by the consumer and is a function of p_1, p_2, V thus:

- demand for good 1 is in direct relationship with the consumer's income;
- demand for good 1 is in an inverse relationship with the price of the good 1;
- demand for good 1 does not depend on the price of the good 2 - 1 and 2 are independent goods.

The optimum quantity consumed of good 2 - and thus the demand function for good 2 - are obtained by replacing equation (6) in (5):

$$q_2^* = \frac{V}{2p_2} = f_2(p_1, p_2, V) \quad (7)$$

The utility maximum (maximum consumer satisfaction) is obtained by replacing the optimal amount consumed expressed by equations (6) and (7) the utility formula:

$$U(q_1^*, q_2^*) = \sqrt{q_1^* q_2^*} = \frac{V}{2\sqrt{p_1 p_2}} = Z(p_1, p_2, V) \quad (8)$$

It is observed that maximum utility is a function of the prices of goods consumed and consumer income.

2. Interpretation of Lagrange multiplier (λ)

Consider a change in consumer income size *d*V. We wonder what impact this change the maximum utility to the consumer, while maintaining constant prices. Changing consumer income lead to changes in optimal quantities consumed in both goods and, consequently, maximum utility obtained by the consumer. Mathematically, the changes are written as follows:

$$p_1 dq_1^* + p_2 dq_2^* = dV \quad \text{(of restriction)}$$

$$\frac{\partial U}{\partial q_1^*} dq_1^* + \frac{\partial U}{\partial q_2^*} dq_2^* = dU(q_1^*, q_2^*) \quad \text{(the total differential of maximum utility function)}$$

We replace the differential total maximum marginal utilities utility function relationships (1) and (2) above:

$$dU(q_1^*, q_2^*) = \lambda p_1 dq_1^* + \lambda p_2 dq_2^* = \lambda\left(p_1 dq_1^* + p_2 dq_2^*\right) = \lambda dV \Rightarrow$$

$$\Rightarrow \frac{dU}{dV} = \lambda$$

When income changes by dV, maximum utility consumer changes by λ.

In conclusion, it Lagrange multiplier reflects how changes in the objective function optimally (usefulness in our case) when of restriction variable (in this case income) is amended by one.

3. Analysis of application functions - elasticity of demand

Definition elasticity of a function *f* to the variable *x*

$$E_{f/x} = \frac{\Delta f}{f} : \frac{\Delta x}{x} = \frac{\Delta f}{\Delta x} \cdot \frac{x}{f} \underset{\Delta x\, mic}{\approx} \frac{\partial f}{\partial x} \cdot \frac{x}{f} \quad (9)$$

Let us apply the definition and approximation (2) if the demand function for the good one of equation (6):

$$E_{q_1^*/p_1} = \frac{\partial q_1^*}{\partial p_1} \cdot \frac{p_1}{q_1^*} = -\frac{V}{2p_1^2} \cdot \frac{p_1}{\frac{V}{2p_1}} = -1$$

The above result indicates that if the price of the goods in January increased by 1%, the quantity demanded of good 1 to fall by about 1%.

It should be noted that if we get $E_{q_1^*/p_1} > 0$, hen one is a good ordinary good otherwise it is a Giffen good.

$$E_{q_1^*/p_2} = \frac{\partial q_1^*}{\partial p_2} \cdot \frac{p_2}{q_1^*} = 0$$

The above result indicates that if the price of the goods in January increased by 1%, the quantity demanded of one good does not change.

It should be noted that if we get $E_{q_1^*/p_2} < 0$, then 1 and 2 are complementary goods and, if $E_{q_1^*/p_2} > 0$, 1 and 2 when goods are substitutable.

$$E_{q_1^*/V} = \frac{\partial q_1^*}{\partial V} \cdot \frac{V}{q_1^*} = \frac{1}{2p_1} \cdot \frac{V}{\frac{V}{2p_1}} = 1$$

The above result indicates that if the price of the goods in January increased by 1%, the quantity demanded of good 1 will increase by about 1%.

It should be noted that if we get $E_{q_1^*/V} > 0$, then the good one is normal and otherwise he is an inferior good.

Summing up the three elasticities get $E_{q_1^*/p_1} + E_{q_1^*/p_2} + E_{q_1^*/V} = -1 + 0 + 1 = 0$.

Economic interpretation of this result is the following: when the prices of both goods increased by 1% and revenue increased by 1%, the amount consumed of one asset does not change.

4. Dual consumer problem

The issue of dual consumer requires minimizing costs in terms of obtaining a utility he desired, namely:

$$\begin{cases} \max\limits_{q_1,q_2} p_1q_1 + p_2q_2 \\ U(q_1,q_2) = \bar{U} \end{cases}$$

where $\bar{U}$ is a utility level of consumer choice.

The dual problem is solved using Lagrange multiplier everything:

$$\min_{q_1,q_2,\lambda} L(q_1,q_2,\lambda) = p_1q_1 + p_2q_2 - \lambda\left(U(q_1,q_2) - \bar{U}\right)$$

First order conditions are written in the same way:

$$\frac{\partial L}{\partial q_1} = 0 \Rightarrow p_1 = \lambda\frac{\partial U}{\partial q_1} \quad (10)$$

$$\frac{\partial L}{\partial q_2} = 0 \Rightarrow p_2 = \lambda\frac{\partial U}{\partial q_2} \quad (11)$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow U(q_1,q_2) = \bar{U} \,(12)$$

Dividing relations (10) and (11) we get the same relationship as in the utility maximization problem.

$$\frac{\partial U}{\partial q_1} : \frac{\partial U}{\partial q_2} = \frac{p_1}{p_2} \Rightarrow \frac{Umg_{q_1}}{p_1} = \frac{Umg_{q_2}}{p_2}$$

For example, the utility function $U(q_1,q_2) = \sqrt{q_1q_2}$ above optimum condition is written:

$$\frac{\frac{\sqrt{q_2}}{2\sqrt{q_1}}}{p_1} = \frac{\frac{\sqrt{q_1}}{2\sqrt{q_2}}}{p_2} \Rightarrow q_2 = \frac{p_1}{p_2}q_1 \;(13)$$

If we replace the quantity consumed of good 2 in the restriction problem of dual achieve optimal amount consumed good one.

$$\sqrt{q_1q_2} = \bar{U} \Rightarrow \sqrt{q_1\frac{p_1}{p_2}q_1} = \bar{U} \Rightarrow q_1^* = \bar{U}\sqrt{\frac{p_2}{p_1}}, q_2^* = \bar{U}\sqrt{\frac{p_1}{p_2}} \;(14)$$

This optimal point is good demand for one type Hicks made by the consumer and is a function of $p_1, p_2, \bar{U}$ thus:

- demand for good 1 is in direct relationship with the consumer fixed utility;

- demand for good 1 is in an inverse relationship with the price of the good one;

- demand for good 1 does not depend on the price of the good 2 - 1 and 2 are independent goods.

Minimum expenditure made by the consumer are obtained by replacing the optimal amount consumed expressed by equations (14) in spending formula:

$$p_1 q_1^* + p_2 q_2^* = p_1 \bar{U} \sqrt{\frac{p_2}{p_1}} + p_2 \bar{U} \sqrt{\frac{p_1}{p_2}} = 2\bar{U}\sqrt{p_1 p_2}$$

This tutorial only introduces the basic concepts of the Langrange multiplier methods. If you are interested, there are many detailed texts on the subject. The goal of this tutorial was to supply some intuition behind the central ideas so that other, more comprehensive and formal sources become more accessible.

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