ANALYTICAL DETERMINATION OF PRESSURE IN THE GUN BARREL TAKING INTO ACCOUNT THE WEAR

KRASIMIR KALEV [1]

Abstract: *A comparative analysis of analytical solution of the inner ballistics problem has been made in this paper. It propounds an analytical solution, which is made by modern software. The results are given in a graphical mode. The process of gun barrel wear and erosion is taken into account by applying a regression analysis. The simulation gives basic ballistics parameters in function of various operation and exploitation factors.*

Key words: *internal ballistics, wearing out*

I. Introduction.

There are a few world scientific trends in the theory of internal ballistics. Each of them creates hypotheses to explain the process of shooting, focusing the attention on physicochemical or physicomechanical processes occurring in the bore of the gun barrel. Despite the differences, the solutions for defining the maximum pressure P_m and muzzle velocity V_{muzzle} for the given loading conditions can be summarized in two groups. The first group (theoretical) takes into account a great part of the ongoing physical, chemical, thermodynamic, gas dynamics and mechanical processes during the shot and is characterized by high accuracy [2, 3, 4, 9]; the other includes empirical calculation systems, which are based on the results of the ballistic tests [7, 10].

In the methods of the theoretical group, possibly a small number of parameters are used, which allows them to be quantified, based on the results of the shooting. On the other hand, the high accuracy of approximation in a limited number of parameters leads to the creation of systems of complex mathematical equations. Despite the difficulties in solving the equations, they are useful for studying the influence of design parameters of the gun barrel on ballistic characteristics, where experienced method is not realizable.

The analytical method for solving the problem of internal ballistics applied independently by two researchers Drozdov [8] and Coppock [1] is considered in this report.

[1] Faculty „Artillery, AAD and CIS”, 1 Carel Shkorpil str.,9700 Shumen, Bulgaria, email: kraskalev@abv.bg

II. Formulation of the problem to determine the pressure of powder gases and the speed of the projectile into the barrel.

1. Assumptions.

Both researchers accept that:

- The powder mass burn rate is proportional to the chamber pressure and the surface area of the powder;
- The surface area of the powder is computed as it burns;
- The powder burns at the breech pressure;
- The pressure gradient between breech and bullet base is computed and assumed to be constant depending on the charge weight to bullet weight ratio;
- The increasing volumes behind the bullet with burning time and bullet movement is taken into account;
- All the energy of an adiabatic expansion is provided by or given to the gas;
- Heat loss to the barrel is estimated and used in the solution;
- Frictional and recoil losses are estimated and used in the solution.

Drozdov took into consideration a part of the "minor works" (for the rotation of the projectile to overcome the friction between the shells, lead belt and the walls of the inner surface of the bore, for moving the gases and unburnt gunpowder and for the recoil of the moving parts), and he accepted that they are proportional to the main operation for the forward movement of the projectile. Also he introduced the concept of "pressure forcing" taking into account the initial resistance to the movement of the projectile.

Coppock evaluated the friction losses of the projectile in the bore of the barrel and moving the recoil parts by increasing the mass of the fictitious projectile to 5%.

Since the shot is a process involving a large number of mechanical, physicochemical, thermal and gas dynamics values (geometric parameters of the bore of the gun barrel, mass and ballistic characteristics of the projectile and charge, temperature and pressure of combustion products, etc. both researchers reduced functional dependence to a system of four equations.

The proposed analytical model includes: the powder burn rate equation, the powder form function, the equation of motion and the energy equation.

2. The powder burn rate equation.

Both authors adopted the same type of the powder burn rate equation.

Drozdov accepted mathematical expression

$$\frac{de}{dt} = u_1 p , \tag{1}$$

where e is the thick layer of burnt gunpowder;
p is the barrel pressure;
u_1 is the burn rate at $p=1$.

Coppock applied the mathematical expression

$$D\frac{df}{dt} = -\beta p , \tag{2}$$

where D *is* the minimum burning dimension of the powder grain;
f *is* the the fraction of the powder grain that is left unburned;
β *is* the linear burn rate of the powder.

3. The powder form function.

Drozdov accepted the expression:

$$\psi = \chi z(1+\lambda z) = \chi z + \chi \lambda z^2 \quad (3)$$

where z is the relative thickness of gunpowder;
λ, χ are the characteristics of the shape of the powder grain.

Coppock used expression for the mass (volume fraction) of the powder burned

$$\varphi = (1-f)(1+\theta f), \quad (4)$$

where θ is the form factor (it can range from -1 to +1).

4. The equation of motion.

In Drozdov it has the form

$$ps = \phi M v \frac{dv}{dl}, \quad (5)$$

where s is the cross-sectional area of the barrel;
l is the path traveled by the projectile in a gun;
M is the weight of the projectile; v – projectile velocity;
ϕ is the adjusted weight of the projectile.

Coppock took into account part of the mass of the gunpowder charge, which also carried out motion forward

$$\left(W_1 + \frac{1}{2}C\right)\frac{dv}{dt} = Ap, \quad (6)$$

where A is the cross-sectional area of the barrel;
W_1 is the adjusted weight of projectile;
C is the powder charge mass.

5. The energy equation.

Using the Resal equation for energy, both Drozdov and Coppock made their multipage very complicated mathematical derivation of the energy equation for the interior ballistics:

- Drozdov in the form

$$ps(l_\psi + l) = fm\psi - \frac{k-1}{2}\varphi M v^2, \quad (7)$$

where l_ψ is the reduced length of free volume of combustion chamber;
k is the ratio of C_p/C_v; C_p is the heat capacity of the gas at constant pressure; C_v is the heat capacity of the gas at constant volume;

- Coppock in the form

$$\varphi\left[1+\frac{p\left(1+\frac{C}{3W_1}\right)}{\lambda\left(1+\frac{C}{2W_1}\right)}\left(\eta-\frac{1}{\delta}\right)\right] = \frac{\bar{\gamma}-1}{2}\left(\frac{W_1}{C}+\frac{1}{3}\right)\frac{V^2}{\lambda} + \frac{Ap(x+l)\left(1+\frac{C}{3W_1}\right)}{C\lambda\left(1+\frac{C}{2W_1}\right)}, \quad (8)$$

where η is the co-volume ideal of the gas;
δ is the density of powder solid;

γ is the ratio of C_p/C_v; C_p is the heat capacity of the gas at constant pressure; C_v is the heat capacity of the gas at constant volume;

l is the reduced length of free volume of combustion chamber at $\varphi=0$.

In this way, the equations which describe the internal ballistic processes derived by Drozdov have the form:

$$\left| \begin{aligned} & u = \frac{de}{dt} = u_1 p \\ & \psi = \chi z(1+\lambda z) = \chi z + \chi \lambda z^2 \\ & ps = \varphi M v \frac{dv}{dl} \\ & ps(l_\psi + l) = f m \psi - \frac{k-1}{2} \varphi M v^2 \end{aligned} \right. , \qquad (9)$$

by Coppock

$$\left| \begin{aligned} & D\frac{df}{dt} = -\beta p \\ & \varphi = (1-f)(1+\theta f) \\ & \left(W_1 + \frac{1}{2}C\right)\frac{dV}{dt} = Ap \\ & \varphi\left[1 + \frac{p\left(1+\frac{C}{3W_1}\right)}{\lambda\left(1+\frac{C}{2W_1}\right)}\left(\eta - \frac{1}{\delta}\right)\right] = \frac{\bar{\gamma}-1}{2}\left(\frac{W_1}{C}+\frac{1}{3}\right)\frac{V^2}{\lambda} + \frac{Ap(x+l)\left(1+\frac{C}{3W_1}\right)}{C\lambda\left(1+\frac{C}{2W_1}\right)} \end{aligned} \right. . \qquad (10)$$

Graphical results of the pressure and velocity as a function of path traveled by the projectile in a gun are made by Matlab and they are shown in figure 1.

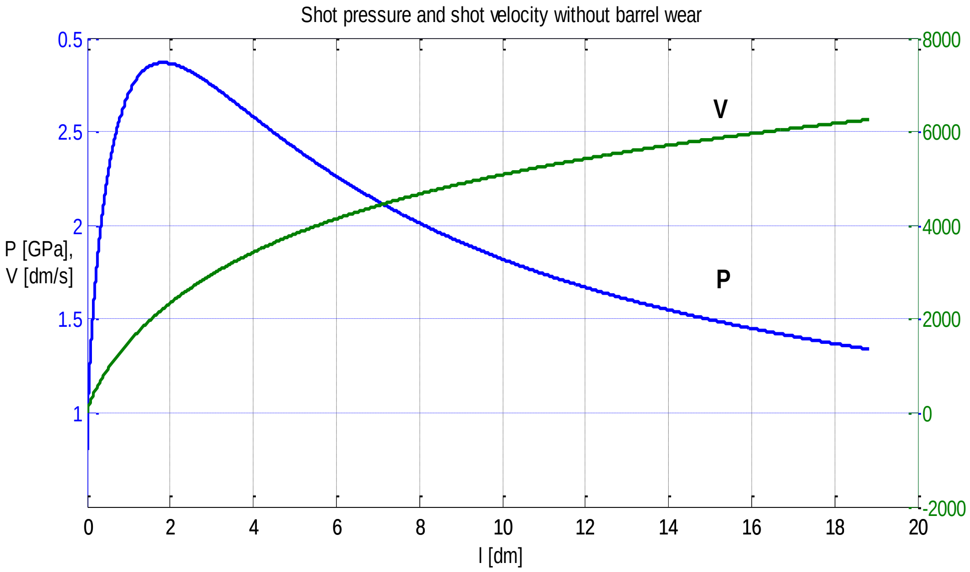

Figure 1.

III. Statistical modeling of the process of wear in the barrel bore leading part at separate charging.

An passive experiment was carried out with 122 mm howitzer, where the geometric parameter is measured – extension of the charging chamber depending on the number of shots.

The process of wear is modeled by applying a regression analysis was performed by standard procedures [11]. Equations (11, 12 and 13) were examined

$$\hat{y}_{x_2} = ax^2 + bx \tag{11}$$

$$\hat{y}_{x_3} = ax^3 + bx^2 + cx \tag{12}$$

$$\hat{y}_{x_5} = ae^{b_1 x} + ce^{dx} \tag{13}$$

Статистическият анализ на регресионните уравнения е извършен с “Statistics Toolbox” на програмния продукт “Matlab”.

The statistical analysis of the regression equations was performed with "Statistics Toolbox" of the software "Matlab".

After solving the system of linear normal equations results of the estimated coefficients of the regression equations are obtained and they are presented in Table 1.

Table 1.

Type of model	Evaluation of coefficients of regression equations			
	a	***b***	***c***	***d***
$\hat{y}_{x_2} = ax^2 + bx$	54.10^{-8}	19.10^{-4}	---	---
$\hat{y}_{x_3} = ax^3 + bx^2 + cx$	$1,23.10^{-10}$	$-7,58.10^{-7-}$	59.10^{-4}	---
$\hat{y}_{x_5} = ae^{b_1 x} + ce^{dx}$	12,77	$2,18.10^{-4}$	-10,98	$3,01.10^{-5}$

The evaluation of the resultant models is based on the following indicators: SSE measures performance according to the sum of squared errors; …….. The results are shown in Table. 2.

Table 2.

Type of model	***SSE***	***R-sguare***	***DFE***	***Adj R- sg***	***RMSE***
$\hat{y}_{x_2} = ax^2 + bx$	211,34	1,0754	15	0,93211	3,5972
$\hat{y}_{x_3} = ax^3 + bx^2 + cx$	96,298	0,8793	14	0,94519	2,8459
$\hat{y}_{x_5} = ae^{b_1 x} + ce^{dx}$	129,58	1,0531	13	0,97652	2,7531

According to the results of the statistical analysis of regression equations, table. 1 and 2 the best performers: SSE, R-sguare; Adj R-sg; RMSE, the best model is the one that has been realized with the regression equation (13) and graphic interpretation presented in Fig. 2.

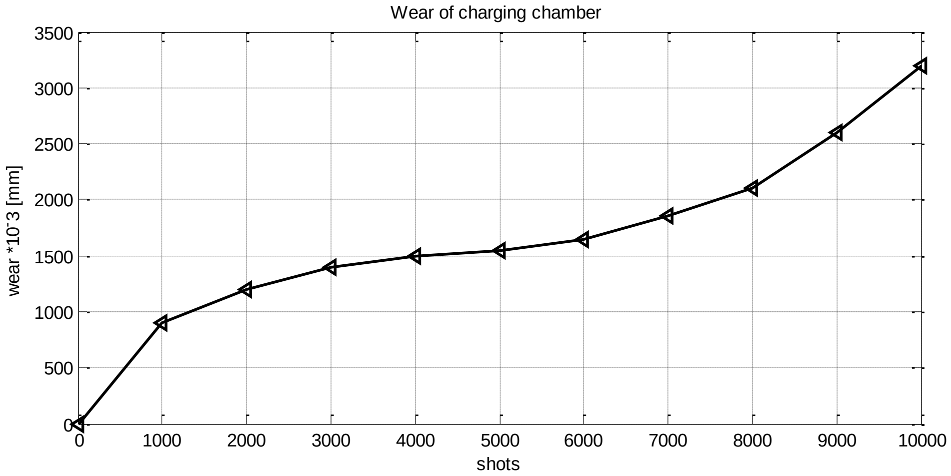

Figure 2. Graphical model of wear of the extension of the charging chamber depending on the shots

III. Solving the problem of internal ballistics.

During the past few decades the field of interior ballistics has been greatly advanced by the development and application of several equations describing mathematical models [6]. The availability of more powerful computers and better computational programs makes it easier and faster to solve problems [4, 5]. All this creates prerequisites for obtaining more accurate results through the application of analytical method to solve the problem of internal ballistics.

In the analytical method of solving the system of equations (9), difficulties arise from the differential equation for the path of the projectile, whose integration leads to complex functions. The method of numerical integration of the system allows solving differential equations at different assumptions about the nature of the combustion charge, tribological processes, the geometry of the bore of the gun barrel, etc., but the variables p and v cannot be determined in advance at different values of l .

Coppock performed classical analytical solution of the system of equations using the approximated formula [1] for determining the path at the time of maximum pressure, decomposition in order of degrees to find the ratio of the maximum pressure to the charge characteristics and the design parameters of the gun barrel.

Drozdov offers a solution for modeling the change of the pressure and velocity of the projectile in the channel of the gun body, consisting of three parts in accordance with the conditional periods of shots which includes more parameters.

From the comparative analysis on the presented functional relationships which describes the single processes going on in the channel of the gun body during the firing, we can conclude that in spite of the large number of approximations and assumptions, they create conditions for clarifying the behavior of ballistic parameters which may be used in contemporary and more accurate methods to solve a number of theoretical and practical problems.

The addition of an additional equation to the system of equations (9) and (10) that takes into account the wear of the channel of the body allows for more precise calculation of inner ballistic parameters of the cannon during the operation.

The resulting solutions make it possible to analyze the change of the pressure of the powder gases, and the speed of the projectile in the function of the distance covered, with a certain probability to predict the state of the geometric dimensions of the inner volume of the gun barrel, to examine the impact of external factors on the change of ballistic characteristics of the cannon, etc.

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